1 A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-2}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\text{m s}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})\,\text{m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

a) $\underline{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ when $t=0$, $\underline{v} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\underline{v} = \underline{u} + \underline{a}t$

$= \begin{pmatrix} 4 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ 1

$= \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

$\underline{v} = 8\underline{i} - 6\underline{j}$ 1

b) when $t=0$, $\underline{r}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\underline{s} = \underline{r} - \underline{r}_0$

$\therefore \underline{r} = \underline{r}_0 + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 4 \\ 0 \end{pmatrix} + \frac{3^2}{2}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ 1

$= \begin{pmatrix} 22 \\ -25/2 \end{pmatrix}$

$\underline{r} = 22\underline{i} - 12.5\underline{j}$ 1

2. A car is initially at rest on a straight horizontal road.

The car then accelerates along the road with a constant acceleration of $3.2\,\text{m}\,\text{s}^{-2}$

Find

(a) the speed of the car after 5 s,

(1)

(b) the distance travelled by the car in the first 5 s.

(2)

(a) $v = u + at$

$v = 0 + 3.2 \times 5$

$v = 16\ \text{ms}^{-1}$ 1

(b) $s = \frac{1}{2}(u+v)t$ ← other suvat equations would also work

$s = \frac{1}{2} \times (0 + 16) \times 5$ 1

$s = 40\text{m}$ 1

3. [*In this question,* **i** *and* **j** *are horizontal unit vectors and position vectors are given relative to a fixed origin O*]

A particle P is moving on a smooth horizontal plane.

The particle has constant acceleration $(2.4\mathbf{i} + \mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, P passes through the point A.

At time $t = 5\,\mathrm{s}$, P passes through the point B.

The velocity of P as it passes through A is $(-16\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$

(a) Find the speed of P as it passes through B.

(4)

The position vector of A is $(44\mathbf{i} - 10\mathbf{j})\,\mathrm{m}$.

At time $t = T$ seconds, where $T > 5$, P passes through the point C.

The position vector of C is $(4\mathbf{i} + c\mathbf{j})\,\mathrm{m}$.

(b) Find the value of T.

(3)

(c) Find the value of c.

(3)

(a) $v = u + at$

$V_B = (-16i - 3j) + (2.4i + j) \times 5$ (1)

$V_B = -4i + 2j$ (1)

speed $= |v|$

$= \sqrt{(-4)^2 + 2^2}$ (1)

$= \sqrt{20}$

$= 2\sqrt{5}$

$= 4.5\ \mathrm{ms^{-1}}$ (1)

start : $A = (44i - 10j)$

end : $C = (4i + cj)$

(b) $s = ut + \frac{1}{2}at^2$

$\underset{\text{end}}{(4i + cj)} = (-16i - 3j)T + \frac{1}{2}(2.4i + j)T^2 + \underset{\text{start}}{(44i - 10j)}$ (1)

i-components : $4 = -16T + 1.2T^2 + 44$ (1)

$1.2T^2 - 16T + 40 = 0$

$T = 10$ or $T = \frac{10}{3}$

$T > 5$ so $T = 10$ seconds (1)

(c) j - components : $C = -3T + \frac{1}{2}T^2 - 10$ (1)
$C = -3(10) + \frac{1}{2}(10^2) - 10$ (1) from part (b)
$C = 10$ (1)

4.

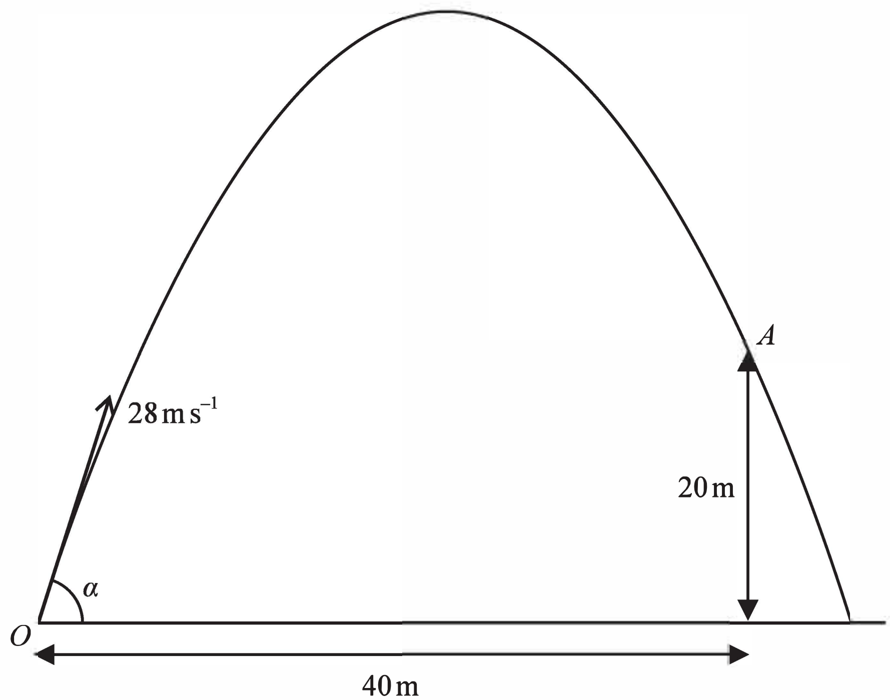


Figure 2

A small ball is projected with speed 28 m s^{-1} from a point O on horizontal ground.

After moving for T seconds, the ball passes through the point A.

The point A is 40 m horizontally and 20 m vertically from the point O, as shown in Figure 2.

The motion of the ball from O to A is modelled as that of a particle moving freely under gravity.

Given that the ball is projected at an angle α to the ground, use the model to

(a) show that $T = \dfrac{10}{7\cos\alpha}$

(2)

(b) show that $\tan^2\alpha - 4\tan\alpha + 3 = 0$

(5)

(c) find the greatest possible height, in metres, of the ball above the ground as the ball moves from O to A.

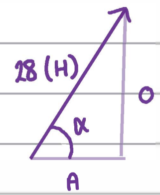
(3)

The model does not include air resistance.

(d) State one other limitation of the model.

(1)

(a)



Initial speed (28) has horizontal and vertical components.

Horizontal (A) $= 28\cos\alpha$
Vertical (o) $= 28\sin\alpha$
} using SOHCAHTOA

Horizontally:

$$28\cos\alpha = \frac{40}{T} \quad (1) \quad \leftarrow \text{speed} = \frac{\text{distance}}{\text{time}}$$

$$28\cos\alpha \times T = 40$$

$$T = \frac{40}{28\cos\alpha}$$

$$T = \frac{10}{7\cos\alpha} \quad (1)$$

(b) Vertically:

$$20 = (28\sin\alpha \times T) + \left(\tfrac{1}{2} \times -g \times T^2\right) \quad (1) \quad \leftarrow s = ut + \tfrac{1}{2}at^2$$

$$20 = (28\sin\alpha \times T) - \tfrac{1}{2}gT^2 \quad (1)$$

$$20 = \left[28\sin\alpha \times \frac{10}{7\cos\alpha}\right] - \left[\frac{1}{2}g\left(\frac{10}{7\cos\alpha}\right)^2\right] \quad (1)$$

$$20 = 40\frac{\sin\alpha}{\cos\alpha} - \frac{g}{2} \times \frac{100}{49\cos^2\alpha}$$

$\dfrac{\sin x}{\cos x} = \tan x$

$$20 = 40\tan\alpha - \frac{100g}{98} \times \frac{1}{\cos^2\alpha}$$

$\dfrac{1}{\cos^2 x} = \sec^2 x$

$\sec^2 x = 1 + \tan^2 x$

$$20 = 40\tan\alpha - \frac{100 \times 9.8}{98} \times (1 + \tan^2\alpha) \quad (1)$$

$$20 = 40\tan\alpha - 10 - 10\tan^2\alpha$$

$$10\tan^2\alpha - 40\tan\alpha + 30 = 0$$

$$\tan^2\alpha - 4\tan\alpha + 3 = 0 \quad (1)$$

(c) $\tan^2\alpha - 4\tan\alpha + 3 = 0$

$(\tan\alpha - 3)(\tan\alpha - 1) = 0$

$\therefore \tan\alpha = 3 \qquad \tan\alpha = 1$ ← select larger value of α to

$\alpha = 71.6° \qquad \alpha = 45°$ (1) obtain "greatest possible height"

$v^2 = u^2 + 2as$ ← at highest point, vertical velocity is 0.

$0 = (28\sin\alpha)^2 + (2\times -g \times H)$ (1)

$0 = (28 \times \sin(71.6°))^2 - 2\times 9.8 \times H$

$0 = 26.56^2 - 19.6H$

$19.6H = 705.43$

$H = 35.99$

$H = 36.0\text{m}$ to 3.s.f (1)

(d) Ball is modelled as a particle. (1)

5.

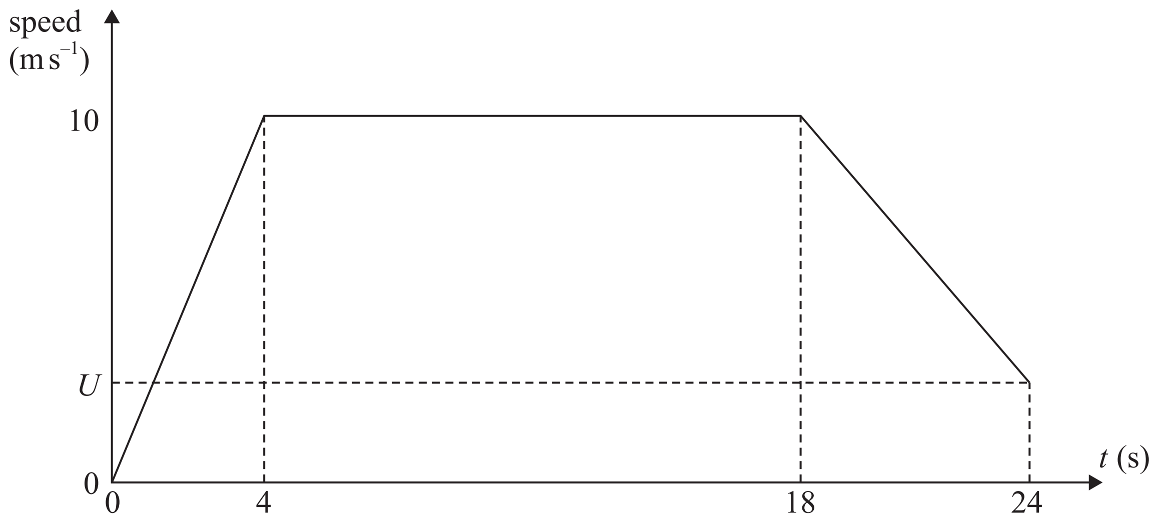


Figure 2

Figure 2 shows a speed-time graph for a model of the motion of an athlete running a **200 m** race in 24 s.

The athlete

- starts from rest at time $t = 0$ and accelerates at a constant rate, reaching a speed of $10 \mathrm{~m~s^{-1}}$ at $t = 4$
- then moves at a constant speed of $10 \mathrm{~m~s^{-1}}$ from $t = 4$ to $t = 18$
- then decelerates at a constant rate from $t = 18$ to $t = 24$, crossing the finishing line with speed $U \mathrm{~m~s^{-1}}$

Using the model,

(a) find the acceleration of the athlete during the first 4 s of the race, stating the units of your answer,

(2)

(b) find the distance covered by the athlete during the first 18 s of the race,

(3)

(c) find the value of U.

(3)

a) in the first 4s,

$v = 10$, $u = 0$, $a = ?$, $t = 4$

$v = u + at$ ⟵ because accelerating at a constant rate

$a = \frac{v}{t} = \frac{10 \text{ ms}^{-1}}{4 \text{ s}}$ (1) $= 2.5 \text{ ms}^{-2}$ (1)

b) distance covered = area under the graph

from $t = 0$ to $t = 4$;

$A = \frac{1}{2} \times 4 \times 10 = 20 \text{ m}$ (1)

from $t = 4$ to $t = 18$;

$A = 10 \times (18 - 4) = 140 \text{ m}$

∴ Total area = total distance covered = $20 + 140$ (1)

$= 160 \text{ m}$ (1)

c) from $t = 18$ to $t = 24$,

athlete decelerates at a constant rate. (can use suvat)

$s = 40$
$u = 10$
$v = U$
$a =$
$t = 6$

total race distance

$s = 200 \text{ m} - 160 \text{ m}$ distance covered from $t=0$ to $t=18$

$= 40 \text{ m}$ (1)

$s = \frac{1}{2}(u + v)t$

$40 = \frac{1}{2}(10 + U)6$ (1)

$\frac{40 \times 2}{6} = 10 + U$

$13\frac{1}{3} - 10 = U$

$U = 3\frac{1}{3}$ (1)